Assignment 4

Prob-1. Refer to Figure 2.2 for the Toluca Company example. A consultant has advised that an increase of one unit in lot size should require an increase of 3.0 in the expected number of work hours for the given production item.

a. Conduct a test to decide whether or not the increase in the expected number of work hours in the Toluca Company equals this standard. Use $\alpha = .05$. State the alternatives, decision rule, and conclusion.

b. Obtain the power of your test in part (a) if the consultant's standard actually is being exceeded by .5 hour. Assume $\sigma\{b_1\} = .35$.

c. Why is $F^* = 105.88$, given in the printout, not relevant for the test in part (a)?

FIGURE 2.2 Portion of MINITAB Regression Output—Toluca Company Example.

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The regression equation is
Y = 62.4 + 3.57 X

Predictor       Coef      Stdev     t-ratio         p
Constant       62.37      26.18        2.38     0.026
X             3.5702     0.3470       10.29     0.000

s = 48.82      R-sq = 82.2%     R-sq(adj) = 81.4%

Analysis of Variance

SOURCE       DF          SS          MS         F         p
Regression    1      252378      252378    105.88     0.000
Error        23       54825        2384
Total        24      307203
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Prob-2. Muscle mass. A person's muscle mass is expected to decrease with age. To explore this relationship in women, a nutritionist randomly selected 15 women from each 10-year age group, beginning with age 40 and ending with age 79. The results follow; X is age, and Y is a measure of muscle mass. Assume that first-order regression model is appropriate.

i:	1	2	3	...	58	59	60
X_i:	43	41	47	...	76	72	76
Y_i:	106	106	97	...	56	70	74

a. Obtain the estimated regression function. Plot the estimated regression function and the data. Does a linear regression function appear to give a good fit here? Does your plot support the anticipation that muscle mass decreases with age?

b. Obtain the following: (1) a point estimate of the difference in the mean muscle mass for women differing in age by one year, (2) a point estimate of the mean muscle mass for women aged $X = 60$ years, (3) the value of the residual for the eighth case, (4) a point estimate of σ^2.

Prob-3. Refer to **Muscle mass Prob-2.**

a. Obtain a 95 percent confidence interval for the mean muscle mass for women of age 60. Interpret your confidence interval.

b. Obtain a 95 percent prediction interval for the muscle mass of a woman whose age is 60. Is the prediction interval relatively precise?

c. Determine the boundary values of the 95 percent confidence band for the regression line when $X_h = 60$. Is your confidence band wider at this point than the confidence interval in part (a)? Should it be?

Prob-4.

Refer to **Muscle mass Prob-2.**

a. Plot the deviations $Y_i - \hat{Y}_i$ against X_i on one graph. Plot the deviations $\hat{Y}_i - \bar{Y}$ against X_i on another graph, using the same scales as in the first graph. From your two graphs, does SSE or SSR appear to be the larger component of $SSTO$? What does this imply about the magnitude of R^2?

b. Set up the ANOVA table.

c. Test whether or not $\beta_1 = 0$ using an F test with $\alpha = .05$. State the alternatives, decision rule, and conclusion.

d. What proportion of the total variation in muscle mass remains "unexplained" when age is introduced into the analysis? Is this proportion relatively small or large?

e. Obtain R^2 and r.

Prob-5.

In a small-scale regression study, five observations on Y were obtained corresponding to $X = 1, 4, 10, 11$, and 14. Assume that $\sigma = .6$, $\beta_0 = 5$, and $\beta_1 = 3$.

a. What are the expected values of MSR and MSE here?

b. For determining whether or not a regression relation exists, would it have been better or worse to have made the five observations at $X = 6, 7, 8, 9$, and 10? Why? Would the same answer apply if the principal purpose were to estimate the mean response for $X = 8$? Discuss.

Prob-6.

Look at the picture. What is wrong in the argument?

$$\sum(Y_i-\hat{Y})^2 = \sum(Y_i-\bar{Y})^2 - \sum(\hat{Y}_i-\bar{Y})^2$$

易知: $E\left(\sum(Y_i-\bar{Y})^2\right) = (n-1)\sigma^2$

$$E\left(\sum(\hat{Y}_i-\bar{Y})^2\right) = \sigma^2 + \beta_1^2\sum(X_i-\bar{X})^2$$

右式取期望 $= (n-2)\sigma^2 + \beta_1^2\sum(X_i-\bar{X})^2$

$$\neq (n-2)\sigma^2$$

这和 Lec3 – Homework 2 是相悖的.